Konrad Polthier is professor of mathematics at Freie Universität Berlin and the DFG research center MATHEON, and member of the Berlin Mathematical School.

Geometry Processing – Discrete Geometry for Virtual Worlds

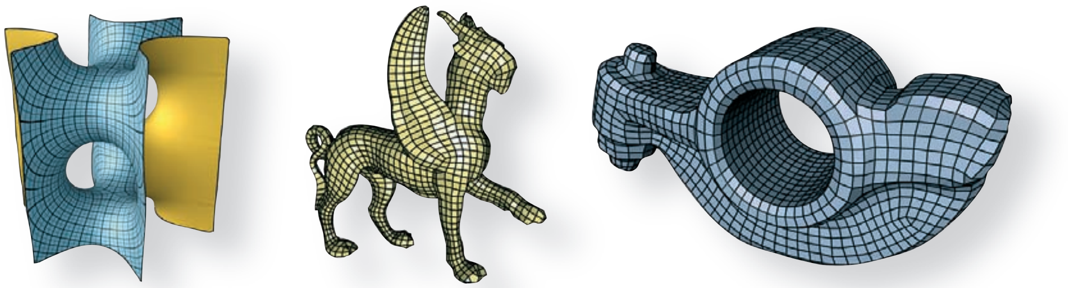

Figure 1 Automatically generated global quadrilateral parametrizations of triangle meshes

Digitizing our 3D world with the same ease, speed and perfection as taking a digital photograph is becoming a key necessity in our daily life. Computer aided design, medical analysis, archeology, animation industry, fashion design, and e-commerce are just a few of the many industrial applications of digital 3D geometries. The new research field *geometry processing* is dedicated to the acquisition, analysis, manipulation, reconstruction and simulation of digital geometries. It interacts with discrete and differential geometry, computer graphics and numerical mathematics.

Digital 3D Models During the past decade we have seen a revolution in 2D image acquisition technologies. The advent of digital cameras in the consumer market has completely changed the photo industry: new image acquisition devices, new image storage formats, new processing software. Beside hardware developments, the main changes in photography are driven by the advances in *digital image processing technology.* Novel algorithms like image compression or filtering techniques such as noise reduction or red eye removal were among the key advances to make digital imaging a consumer product and create a completely new industry.

Now we are stepping from 2D images into the world of 3D geometries, and see a similar revolution. Novel acquisition technologies such as 3D laser scanning and tracking, computer tomography, and magnetic resonance imaging have reached a high degree of perfection and provide an instant availability of high quality 3D digital data. *Geometry processing* researches the whole engineering pipeline including data acquisition, noise removal, surface reconstruction, structural segmentation, shape optimization, or model compression. In this sense, geometry processing takes image processing to 3D.

Discrete Differential Geometry The basic building blocks of digital geometries are points, triangles, tetrahedra and other primitive entities. Mathematically, digital models are often described as simplicial complexes or cell complexes in the language of algebraic topology, or as finite element meshes in the language of numerical mathematics, or arise as stochastic geometries. The discrete and often non-differentiable nature prevents a direct application of classic differential geometric methods. *Discrete differential geometry* is a novel research field providing a mathematical framework that extends classic differential geometry to polyhedral meshes and other non-differentiable shapes. For example, it develops curvature operators for non-differentiable simplicial surfaces which fully rely on the sole information of the discrete model, i.e. no approximation with

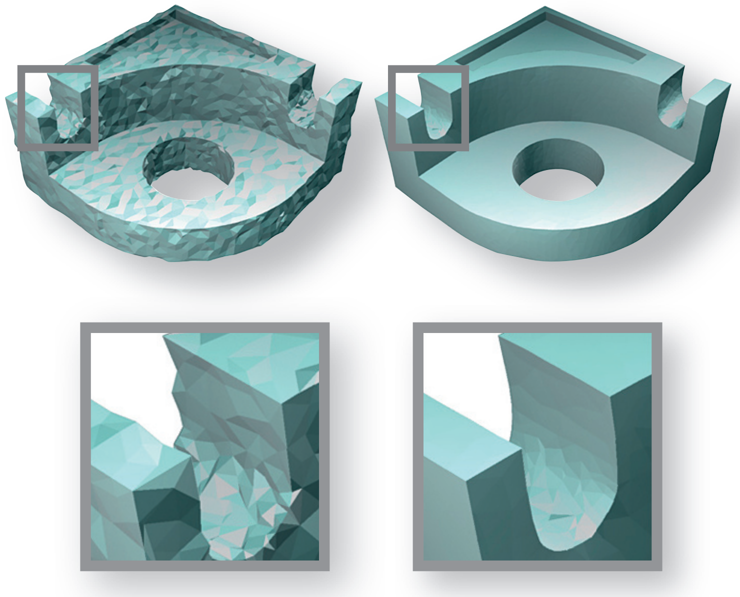

Figure 2 Anisotropic smoothing with feature preservation of a noisy CAD model based on a discrete shape operator with smallest support

Images: B. Wannenmacher, F. Kälberer, M. Nieser, K. Hildebrandt, B. Janzen

smoother shapes is needed. A major goal of discrete geometry is the finding of new geometric invariants which are kept during geometry processing applications, and thereby reduce approximation errors.

Discrete differential geometry comes in different flavors, from piecewise linear and polynomial techniques with close contact to finite element techniques, to circle packings, or even point set models with contact to stochastic geometry.

Reverse Engineering Many of the most active and fruitful ongoing research activities in geometry processing are lined up along the *reverse engineering pipeline* which aims at the automatic digitization of physical shapes into optimal virtual models. Sampling and digitization, meshing and noise removal, segmentation and feature extraction, mesh compression, numerical simulations, and manufacturing planning are key research issues. Discrete differential geometry plays a central role in the discovery and formalization of the invariants of discrete geometries including dynamic shapes and physical properties.

Virtual 3D Worlds At least since the recent hype about the online world *Second Life*, everyone can imagine the potential of immersive 3D worlds. Countries may have virtual embassies, companies trade virtual products through virtual offices, and individuals own virtual real-estate – just as in real life. Besides artificial worlds, numerous activities aim at 3D digital versions of basically all shapes seen in our real world. From consumer products to architectural buildings, from humans, animal and plants to natural landscapes, finally realistic digitial models of all 3D shapes are needed, possibly as dynamic shapes with physical properties. These applications and the ever increasing richness of detail generate an enormous challenge for the development of most effective data structures, network capacities and algorithms for numerical simulations.

Digital Shape Repositories Visualization in geometry started long before the advent of computer technologies. In the 19th century Felix Klein together with Hermann Amandus Schwarz pushed the development of plaster models of important geometric shapes for educational purposes. One of their producers, Martin Schilling, sold plaster models world wide, from Göttingen, to Tokyo, to Urbana-Champaign (the largest collection I have seen outside of Germany). Klein showcased a collection of mathematical models at the World's Columbian Exposition 1893 in Chicago. Nowadays many plaster models are covered with dust, but more and more are being exhibited again. As a continuation of Klein's vision we founded the electronic journal EG-Models **www.eg-models.de** for the publication of digital geometry models for research and education. Its submissions are peer-reviewed and listed in Zentralblatt. A similar database maintained by the aim@shape project at **dsw.aimatshape.net** showcases well-known benchmark models for geometry processing.

MESH – the film Although discrete geometry is a new fascinating research topic – it has been around for years, and its origins date back to the mathematics in ancient Greece. Plato, who lends his name to the five regular Platonic solids, considered the world as being entirely built out of triangles. In his treatise *Timaeus* Plato considered the five regular solids, the cube, the tetrahedron, the octahedron, the icosahedron and the dodecahedron as the fundamental elements of nature: earth, fire, air, water and the universe, quintessence. He reasoned, when each element can be constructed from triangles then everything in existence must be based on triangles. We may smile about Plato's ideas but when looking at computer graphics and numerical mathematics we again see our world as being entirely built out of triangles and other finite elements. The internationally awarded video MESH **www.mesh-film.de** (joint with Beau Janzen) is an entertaining journey through discrete geometry from Plato and Euclid via Kepler, Newton and Leibniz to modern real-world applications.

Challenge The scientific depth and diversity of problems in geometry processing provide an excellent chance for new interdisciplinary mathematical research collaborations and for new cooperations with a variety of industrial partners.

Figure 3 Scene on Plato's Timaeus from video MESH