

Hans-Christian Hege  
Konrad Polthier (Editors)

# Visualization and Mathematics III

With 240 Figures, 47 in Color



Springer

Hans-Christian Hege  
Konrad-Zuse-Zentrum  
für Informationstechnik Berlin (ZIB)  
Takustraße 7  
14195 Berlin, Germany  
e-mail: hege@zib.de

Konrad Polthier  
Institut für Mathematik, MA 8-3  
Technische Universität Berlin  
Straße des 17. Juni 136  
10623 Berlin, Germany  
e-mail: polthier@math.tu-berlin.de

Cover image: Stereographic projection of a compact minimal surface in  $S^3$   
by H.B. Lawson. By K. Polthier (TU-Berlin).

Cataloging-in-Publication Data applied for

A catalog record for this book is available from the Library of Congress.

Bibliographic information published by Die Deutsche Bibliothek  
Die Deutsche Bibliothek lists this publication in the Deutsche Nationalbibliografie;  
detailed bibliographic data is available in the Internet at <http://dnb.ddb.de>

Mathematics Subject Classification (2000): 68U05, 68U10, 68U20; 53-04, 65D18, 65D17,  
65S05, 97-04, 97U70

ISBN 3-540-01295-8 Springer-Verlag Berlin Heidelberg New York

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilm or in any other way, and storage in data banks. Duplication of this publication or parts thereof is permitted only under the provisions of the German Copyright Law of September 9, 1965, in its current version, and permission for use must always be obtained from Springer-Verlag. Violations are liable for prosecution under the German Copyright Law.

Springer-Verlag Berlin Heidelberg New York  
a member of BertelsmannSpringer Science+Business Media GmbH

<http://www.springer.de>

© Springer-Verlag Berlin Heidelberg 2003  
Printed in Germany

The use of general descriptive names, registered names, trademarks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

Typeset in  $\text{\TeX}$  by the authors  
Cover design: *design & production* GmbH, Heidelberg

Printed on acid-free paper 46/3142db - 5 4 3 2 1 0 -

# Preface

Mathematical Visualization aims at an abstract framework for fundamental objects appearing in visualization and at the application of the manifold visualization techniques to problems in geometry, topology and numerical mathematics. The articles in this volume report on new research results in this field, on the development of software and educational material and on mathematical applications.

The book grew out of the third international workshop “Visualization and Mathematics”, which was held from May 22-25, 2002 in Berlin (Germany). The workshop was funded by the DFG-Sonderforschungsbereich 288 “Differential Geometry and Quantum Physics” at Technische Universität Berlin and supported by the Zuse Institute Berlin (ZIB) and the DFG research center “Mathematics for Key Technologies” (FZT 86) in Berlin. Five keynote lectures, eight invited presentations and several contributed talks created a stimulating atmosphere with many scientific discussions.

The themes of this book cover important recent developments in the following fields:

- Geometry and Combinatorics of Meshes
- Discrete Vector Fields and Topology
- Geometric Modelling
- Image Based Visualization
- Software Environments and Applications
- Education and Communication

We hope that the research articles of this book will stimulate the readers’ own work and will further strengthen the development of the field of Mathematical Visualization.

We appreciate the thorough work of the authors and reviewers on each of the individual articles, and we thank you all. Beside the editors, the reviewers and members of the program committee were:

Helmut Alt	Ulrich Kortenkamp
Tom M. Apostol	Jens-Peer Kuska
James Arvo	Carsten Lange
Chandrajit Bajaj	Gregory Leibon
Thomas Banchoff	Nelson L. Max
Philippe Bekaert	Heinrich Müller
Werner Bengler	Gregory M. Nielson
Alexander Bobenko	Ronny Peikert
Alexander Bogomjakov	Ulrich Pinkall
Philip L. Bowers	Helmut Pottmann
Ken Brakke	Jürgen Richter-Gebert
Claude Bruter	Martin Rumpf
Matthieu Desbrun	Dietmar Saupe
Peter Deuffhard	Roberto Scopigno
Thomas Ertl	Hans-Peter Seidel
Gerald E. Farin	James Sethian
George Francis	Marc Stamminger
Hans Hagen	John Sullivan
Andrew J. Hanson	Nobuki Takayama
Joel Hass	Gabriel Taubin
David Hoffman	Daniel Weiskopf
Victoria Interrante	Rüdiger Westermann
Chris Johnson	Ross Whitaker
Michael Joswig	Luiz Velho
Alexander Keller	Jarke van Wijk
Leif Kobbelt	Günter M. Ziegler

Special thanks to Robert Staufenbiel for his help in compiling the manuscripts and creating the LaTeX source of this book.

Berlin, 2002  
 Hans-Christian Hege  
 Konrad Polthier

# Table of Contents

Preface .....	V
---------------	---

---

## Part I Geometry and Combinatorics of Meshes

---

### Planar Conformal Mappings of Piecewise Flat Surfaces 3

*Philip L. Bowers, Monica K. Hurdal*

1 An Inversive Distance Primer .....	6
2 Piecewise Flat Surfaces and Circle Packings .....	12
3 Hexagonal Refinement .....	15
4 Proving Convergence and Conformality .....	17
5 A Gallery of Quadrilaterals .....	23
6 Implementation: Practical Experimental, Computational, and Theoretical Issues .....	31

### Discrete Differential-Geometry Operators for Triangulated 2-Manifolds 35

*Mark Meyer, Mathieu Desbrun, Peter Schröder, Alan H. Barr*

1 Introduction .....	35
2 Defining Discrete Operators .....	38
3 Discrete Mean Curvature Normal .....	40
4 Discrete Gaussian Curvature .....	45
5 Discrete Principal Curvatures .....	46
6 Results and Applications .....	49
7 Discrete Operators in $nD$ .....	53
8 Conclusion .....	54

### Constructing Circle Patterns Using a New Functional 59

*Boris A. Springborn*

1 Introduction .....	59
2 Circle Patterns .....	60
3 Preliminaries .....	61
4 The Functional .....	62
5 Examples .....	64
6 Numerical Evaluation of the Functional .....	66

<b>Constructing Hamiltonian Triangle Strips on Quadrilateral Meshes</b>	69
<i>Gabriel Taubin</i>	
1 Introduction . . . . .	70
2 Graphs and Meshes . . . . .	71
3 Hamiltonian Paths . . . . .	74
4 Basic Algorithm . . . . .	77
5 Efficient Implementation . . . . .	78
6 Diagonal Graph Structure . . . . .	79
7 Transparent Vertex Caching . . . . .	83
8 Implementation and Complexity . . . . .	86
9 Subdivision . . . . .	87
10 Borders . . . . .	88
11 Conclusions and Future Work . . . . .	89

---

## Part II Discrete Vector Fields and Topology

---

<b>Visualizing Forman's Discrete Vector Field</b>	95
<i>Thomas Lewiner, Helio Lopes, Geovan Tavares</i>	
1 Introduction . . . . .	95
2 Basic Concepts . . . . .	96
3 Hypergraphs and Hypertrees . . . . .	99
4 Algorithm . . . . .	101
5 Applications . . . . .	106
6 Future Works . . . . .	109

<b>Identifying Vector Field Singularities Using a Discrete Hodge Decomposition</b>	113
<i>Konrad Polthier and Eike Preuß</i>	
1 Introduction and Related Work . . . . .	113
2 Setup . . . . .	116
3 Discrete Rotation . . . . .	117
4 Discrete Divergence . . . . .	120
5 Hodge Type Decomposition of Vector Fields . . . . .	123
6 Decomposition Algorithm and Detecting Vector Field Singularities . . . . .	127
7 Examples . . . . .	129
8 Conclusions and Future Work . . . . .	130

<b>Searching for Knotted Spheres in 4-dimensional Space</b>	135
<i>Dennis Roseman</i>	
1 Background for Random Knots . . . . .	135
2 Generating Random Collections of Objects in $\mathbb{R}^n$ . . . . .	137
3 Methods of Generating Random Spheres in $\mathbb{R}^4$ . . . . .	138
4 Random Icosahedra in $\mathbb{R}^4$ . . . . .	141

5	Sorting Knottings into Equivalence Classes .....	144
6	The Role of Visualization.....	148
7	Ordering Knots .....	148

### **3D Loop Detection and Visualization in Vector Fields** 151

*Thomas Wischgoll, Gerik Scheuermann*

1	Introduction .....	151
2	Mathematical Background.....	152
3	Loop Detection .....	153
4	Results.....	158

---

## **Part III Geometric Modelling**

---

### **Minkowski Geometric Algebra and the Stability of Characteristic Polynomials** 163

*Rida T. Farouki, Hwan Pyo Moon*

1	Introduction .....	163
2	Minkowski Geometric Algebra .....	164
3	Families of Curves and Envelopes.....	168
4	Stability of Characteristic Polynomials .....	170
5	Complex Disk Polynomials .....	173
6	Hurwitz Stability of Disk Polynomials.....	175
7	$\Gamma$ -Stability of Disk Polynomials .....	179
8	Robustness Margin of Disk Polynomials .....	183
9	Closure .....	185

### **Subdivision Invariant Polynomial Interpolation** 189

*Stefanie Hahmann, Georges-Pierre Bonneau, Alex Yvart*

1	Introduction .....	189
2	Related Works .....	190
3	Subdivision Invariance .....	191
4	Subdivision Invariant $G^1$ Polynomial Triangular Interpolant .....	193
5	Results.....	197
6	Conclusion .....	198

### **Another Metascheme of Subdivision Surfaces** 201

*Heinrich Müller, Markus Rips*

1	Introduction .....	201
2	Elementary Schemes Based on Vertex Assignment .....	203
3	Inversion .....	209
4	Selected Composed Subdivision Schemes .....	212
5	Subdivision Schemes of Higher Order .....	216
6	Concluding Remarks .....	219

<b>Geometry of the Squared Distance Function to Curves and Surfaces</b>	221
<i>Helmut Pottmann, Michael Hofer</i>	
1 Introduction . . . . .	221
2 Graph Surface of the Squared Distance Function to a Planar Curve . . . . .	223
3 Quadratic Approximations to $d^2$ . . . . .	226
4 Squared Distance Function to a Surface and its Second Order Approximants . . . . .	231
5 Squared Distance Function to a Space Curve . . . . .	234
6 Application to Geometric Optimization Problems . . . . .	235
7 Future Research . . . . .	240

---

## Part IV Image Based Visualization

---

<b>A Multiscale Fairing Method for Textured Surfaces</b>	245
<i>Ulrich Clarenz, Udo Diewald, Martin Rumpf</i>	
1 Introduction . . . . .	245
2 Image Processing Background . . . . .	246
3 Anisotropic Geometric Diffusion . . . . .	247
4 Coupling Anisotropic Texture and Surface Diffusion . . . . .	250
5 Comparison and Conclusions . . . . .	255
<b>Generalized Block Iterative Methods</b>	261
<i>Michel Leblond, François Rousselle, Christophe Renaud</i>	
1 Introduction . . . . .	261
2 Generalized Block Partitioning of a Matrix . . . . .	265
3 Generalized Block Iterative Methods . . . . .	267
4 Application to Radiosity . . . . .	273
5 Conclusion and Perspectives . . . . .	280
6 Acknowledgements . . . . .	283
<b>Fast Difference Schemes for Edge Enhancing Beltrami Flow and Subjective Surfaces</b>	287
<i>Ravi Malladi, Igor Ravve</i>	
1 Introduction . . . . .	287
2 Implicit Scheme for Beltrami Flow . . . . .	289
3 Simulation Results for Beltrami Flow . . . . .	292
4 Implicit Scheme for Subjective Surfaces . . . . .	294
5 Simulation Results for Completing Missing Boundaries . . . . .	296
6 Closing Remarks . . . . .	296



---

**Part V Software Environments and Applications**


---

<b>ALICE on the Eightfold Way: Exploring Curved Spaces in an Enclosed Virtual Reality Theater</b>	305
<i>George K. Francis, Camille M.A. Goudeseune, Henry J. Kaczmarek, Benjamin J. Schaeffer, John M. Sullivan</i>	
1 Introduction . . . . .	305
2 Other Fully Enclosed Virtual Reality Theaters . . . . .	307
3 Cluster Architecture . . . . .	307
4 Mathematical Visualization of Three-dimensional Geometries . . . . .	310
<b>Computation and Visualisation in the NUMLAB Numerical Laboratory</b>	317
<i>Joseph M.L. Maubach, Alexandru C. Telea</i>	
1 Introduction . . . . .	317
2 The Mathematical Framework . . . . .	321
3 The Software Framework . . . . .	322
4 An Efficient NUMLAB Finite Element Implementation . . . . .	325
5 Application Design and Use . . . . .	331
6 Conclusions and Future Work . . . . .	333
<b>A Generic Programming Approach to Multiresolution Spatial Decompositions</b>	337
<i>Vinícius Mello, Luiz Velho, Paulo Roma Cavalcanti, Cláudio T. Silva</i>	
1 Introduction . . . . .	337
2 Background . . . . .	338
3 Concepts . . . . .	339
4 Models . . . . .	350
5 Applications . . . . .	351
6 Conclusion . . . . .	355
<b>Mathematical Modelling and Visualisation of Complex Three-dimensional Flows</b>	361
<i>Alfio Quarteroni, Marzio Sala, M.L. Sawley, N. Parolini, G. Cowles</i>	
1 Introduction . . . . .	361
2 Mathematical Formulation . . . . .	364
3 Complexity . . . . .	366
4 Visualisation Techniques . . . . .	370
5 Conclusions . . . . .	373
<b>webMathematica</b>	379
<i>Tom Wickham-Jones</i>	
1 Introduction . . . . .	379
2 webMathematica Technology . . . . .	380
3 Web Extension Technologies . . . . .	382

4	web <i>Mathematica</i> Services.....	383
5	Summary.....	389
6	References.....	390

---

## Part VI Education and Communication

---

### **Films: A Communicating Tool for Mathematics** 393

*Michele Emmer*

1	Introduction.....	393
2	Mathematics: a Special Case?.....	395
3	The Mathematics and Art Project.....	396
4	Mathematics and Fiction Films.....	401
5	Final Comments.....	403

### **The Potentials of Math Visualization and their Impact on the Curriculum** 407

*Beau Janzen*

1	Introduction.....	407
2	Palpability.....	409
3	Context.....	409
4	Implementation.....	412
5	Impact on the Nature of the Curriculum.....	416
6	Impact on the Content of the Curriculum.....	419
7	Conclusion.....	420

### **Appendix: Color Plates**..... 423

### **Index**..... 453