Guest Editors' Introduction: Special Section on Mathematics and Visualization

Hans-Christian Hege and Konrad Polthier

Welcome to the special section on Mathematics and Visualization. Mathematics is becoming an increasingly important tool in visualization, aiding the development and analysis of efficient algorithms and data structures. As visualization is beginning to evolve into a mature science and the underlying technology is becoming increasingly complex, the need for a rigorous framework is becoming obvious. Formal mathematical language provides a solid ground which allows us to embed visualization problems within an abstract setting and thereby to make well-established analytical tools applicable. For example, convergence issues and the numerical stability of visualization algorithms are essential for effective and reliable results in visualization.

The editors of this special section have actively supported the convergence of mathematics and visualization as organizers of the international conference series Visualization and Mathematics since 1995, the film festival VideoMath at the International Congress of Mathematicians ICM 1998, as well as at various international workshops and summer schools. These activities helped to establish a small but worldwide community of researchers who perform work in this border area; they also gave rise to the new book series Mathematics + Visualization and the videoseries VideoMATH published by Springer Verlag.

In particular, the international conference series Visualization and Mathematics in Berlin intensified the interaction of mathematicians and scientists from computer graphics and visualization. The current special section is comprised of a set of five carefully selected articles which are elaborated publications of the latest conference in 2002. In order to emphasize the interdisciplinary character of the young scientific field of *mathematical visualization*, the articles in this special section were selected to cover a broad range of topics from theory to praxis. The papers include topics from discrete and differential geometry, combinatorial and differential topology, graph theory, vector field analysis, geometry compression, image processing, signal processing, feature detection, image segmentation, numerical methods, and partial differential equations.

The articles were selected by the editors of this special section and each carefully reviewed by at least three referees. In the following, we summarize the articles.

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"Applications of Forman's Discrete Morse Theory to Topology Visualization and Mesh Compression" by Thomas Lewiner, Héio Lopes and Geovan Tavares: Many advances in science arise from the combination of different fields. Lewiner et al. relate Forman's discrete geometry, a purely mathematical construction, with popular mesh compression schemes, such as Edgebreaker. This link unveils a new topological view on compression algorithms. The introduction of Forman's discrete Morse theory and vector fields provides a new tool for topology visualization and the understanding of visualization algorithms.

"Visualizing a Sphere Eversion" by George Francis and John M. Sullivan: Finding a process to turn a sphere inside out has been an active research area for geometers and topologists for more than 50 years. Mathematical visualization—using computer graphics tools, numerical algorithms, and geometric invention—provides an adequate collection of techniques to calculate and visualize the complex deformations of the eversion process. Here, mathematical visualization provides insight into abstract constructions and explains mathematics to a broader audience. The paper by Francis and Sullivan provides a glimpse of the exciting history of the sphere eversion problem and presents the underlying techniques of the author's awarded computer graphics animation "Optiverse."

"Robust Feature Detection and Local Classification for Surfaces Based on Moment Analysis" by Ulrich Clarenz, Martin Rumpf, and Alex Telea: Geometry analysis and processing has become a major topic in computer graphics, as well as feature detection in visualization. The authors present a surface classification method for triangulated surfaces, with applications in surface fairing, mesh decimation, surface segmentation, and surface matching. The method uses local zero and first order moments as an alternative to curvature and utilizes structure tensor-like concepts. The method is well-founded, explained in detail, and shows convincing performance under various conditions. It provides an indicator for the smoothness of a given discrete surface and comes together with a built-in multiscale. Interesting aspects are the difference between the computation of moments in the smooth and nonsmooth surface as well as the analysis of how the moments scale with the scale parameter for smooth and nonsmooth areas.

"Fast Evolution of Image Manifolds and Application to Filtering and Segmentation in 3D Medical Images" by Thomas Deschamps, Ravi Malladi, and Igor Ravve: Image analysis techniques are interesting in visualization as they represent "feature detection" methods for one major data type, namely, images. Additionally, many of the well-advanced techniques can be modified for other kinds of data. In the last few years, image processing techniques based on partial differential equations have become an important new area. However, from a practical point of view, such methods typically are time consuming. This paper elaborates on numerical techniques for fast computation of Beltrami flow, one of the most effective denoising techniques in image processing. In addition the proposed numerical techniques are adapted and applied to subjective surface computation.

"Crest Lines for Surface Segmentation and Flattening" by Georgios Stylianou and Gerald Farin: This paper presents a method for extracting feature curves called crest lines from a triangulated surface. From the crest lines, a geodesic Voronoi diagram is computed to segment the surface into several regions. Using theory from graph embeddings, based on the geodesic Voronoi diagram, a fast surface flattening algorithm is proposed. It is demonstrated how these techniques can be successfully applied to cortical surfaces of human brains.

We thank all of the authors for their contributions, their enthusiasm, and their energy. We are particularly grateful to the referees for their detailed reviews and insightful comments which strongly supported the evolution of the papers. We also thank the editorial board of the *IEEE Transactions on Visualization and Computer Graphics* for their support and patience.

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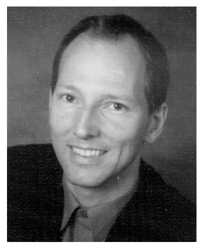
Guest Editors

Hans-Christian Hege is head of the Scientific Visualization Department at Zuse Institute Berlin (ZIB). After studying physics and mathematics, he performed research in computational physics and quantum field theory at Freie Universität Berlin (1984-1989). Then, he joined ZIB, initially as a scientific consultant for high performance computing, then as head of the Scientific Visualization Department, which he started in 1991. His group performs research in data visualization and creates visualization software, e.g., the Amira software. He is also cofounder of mental images (1986) and of Indeed-Visual Concepts GmbH (1999), which he built up as CEO (-2003) and for which he now consults as a scientific advisor. He served as a program committee member for IEEE Visualization (VIS), Eurographics-IEEE Symposium on Visualization (VisSym), the Eurographics Symposium on Geometry Processing (SGP), and the Symposium on Point Based Graphics (PGB). He regularly lectures at Universitat Pompeu Fabra, Barcelona (Spain) and the German Film School (University for Digital Media Production) in Elstal, where he became an honorary professor of scientific visualization in 2003. His current research interests are in visual data analysis and computer graphics as well as applications in natural and life sciences.



Konrad Polthier received the PhD degree in mathematics from the University of Bonn and the Habilitation from Technische Universität Berlin with work on "Polyhedral Surfaces of Constant Curvature." He is the scientist in charge of the key area "Visualization" at the DFG Research Center "Mathematics for key technologies" and head of the research group "Geometry and Visualization" at Zuse Institute Berlin (ZIB) (since 2004). His current interests are discrete and differential geometry, applied geometry, and mathematical visualization. He is on the editorial board of five mathematics journals, book, and video series. He has published several books, coauthored the award-winning mathematical software packages JavaView (www.javaview.de) and Grape (European Academic Software Award in Mathematics), and produced computer generated mathematical videos. His videos were shown at the SIGGRAPH Electronic Theatre and many other juried venues, including VideoMath at ICM '98, Sibgrapi, and NICOGRAPH.